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Phase-angle measurements between hot-wire signals in the turbulent wake of a two-dimensional bluff body

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Measurements have been made of the phase difference between the signals of two hot wires separated in the stream direction and placed in and close to the turbulent wake of a two-dimensional bluff body. Results are presented in the form of the wavelength and the phase speed of perturbations at the vortex shedding frequency for various positions in the flow. It is shown that values of the phase speed may be obtained that are in agreement with published values of the vortex transport velocity. It is necessary to ensure that the hot wires are close to the paths of the centres of the vortices in the wake, otherwise spurious results caused by the influence of the bound vorticity at the model will be obtained.

1. Introduction

Investigations of the vortex wakes of two-dimensional bodies have frequently included measurements of vortex longitudinal spacing. The usual technique has been to measure the correlation coefficient between the signals of two hot-wire anemometers separated in the wake in the direction of mean flow. One wire is held stationary and the other traversed up- and downstream. The correlation coefficient varies periodically with the distance between the wires and its wavelength is taken to be the vortex longitudinal spacing a. The longitudinal spacing derived in this way is thus dependent on measurements made sufficiently far downstream for a to be nearly constant over the stations at which the correlation coefficient is measured.

The velocity U_N with which vortices in a vortex wake are transported downstream relative to the body is less than the free-stream velocity U_0. U_N is given by

$$U_N = Na, \tag{1}$$

where N is the frequency of vortex shedding from one side of the body. The aim of the present work was to develop a system for finding U_N which was sufficiently localized that it might be useful in examining the region close to the body where the vortices are accelerating away from the formation region. Corresponding values of a could then be found from U_N using (1). The method that was used is similar to one introduced by Stegen & Van Atta (1970) for measuring phase speeds in grid turbulence. Two hot-wire anemometer probes were mounted together a fixed distance apart in a holder which could be traversed anywhere in the flow. The anemometer signals were recorded on magnetic tape and subsequently digitally analysed to find the phase difference between them and the vortex shedding frequency. The phase difference $\theta_{rs}(f)$ between the signals of two sensors in the flow is a function of the frequency f. The wavelength $\lambda(N)$ of a perturbation at the shedding frequency travelling between the hot wires is given by

$$\lambda(N) = 2\pi l/\theta_{rs}(N), \tag{2}$$

where l is the hot-wire spacing in the direction of the mean flow. The value of $\theta_{rs}(N)$ obtained is only significant if the coherence between the signals at frequency N is close to unity. For the results presented below, the coherence $\gamma_{rs}(N)$ at the shedding frequency was always greater than 0·98. If the coherence is high and the perturbation at the shedding frequency is solely due to the passage of the vortices then $\lambda(N)$ is a measure of the vortex longitudinal spacing a. Under these same conditions, the phase speed $C(N)$ at the shedding frequency, which is given by

$$C(N) = 2\pi lN/\theta_{rs}(N), \tag{3}$$

is a measure of the vortex transport velocity U_N. Bloor & Gerrard (1966) measured the phase angles between the signals of hot wires in the wake of a circular cylinder. Their means of measuring the phase angle was an analog phasemeter and because of turbulence in the wake and the rapid reduction in strength of the periodic fluctuations outside it, they were only able to obtain readings on a line between 1·4 and 1·6 cylinder diameters from the wake centre-line. In the experiments described below, because the hot-wire signals were analysed digitally, measurements of the phase difference could be made at positions inside and at some distance from a turbulent vortex wake. The computer programs used to perform the analyses are described by Kinns (1973).

2. Experimental arrangement

The experiments were carried out in the 510×720 mm rectangular working section of an open-return wind tunnel in the Aerodynamics Laboratory of the Cambridge University Engineering Department. Measurements were made in the turbulent wake of the D-shaped body shown in figure 1, which spanned the shorter dimension of the working section. The co-ordinate system is also shown in figure 1. The cross-section of the body was a semi-circle upstream of a square and the base width h of the model was 25·4 mm. Experiments were performed at a Reynolds number based on h of $1{\cdot}5 \times 10^4$ and, unless the contrary is stated, were carried out in the plane of the midspan of the model ($z = 0$). Although end-plates were not fitted to the model it was found that the base pressure, measured with tappings recessed along the centre-line of the model base, was uniform across most of the span. The base pressure rose slightly within one base width of each end of the model.

DISA constant-temperature hot-wire anemometers were used to measure velocity fluctuations in the flow. Two DISA 55A25 miniature probes were mounted in an insulated clamp on a traverse gear which enabled them to be moved anywhere in the flow. The orientation of the probes is shown in figure 2, where $l_0 = 0{\cdot}6h$ and $z_0 = 0{\cdot}1h$. These values were a compromise between the

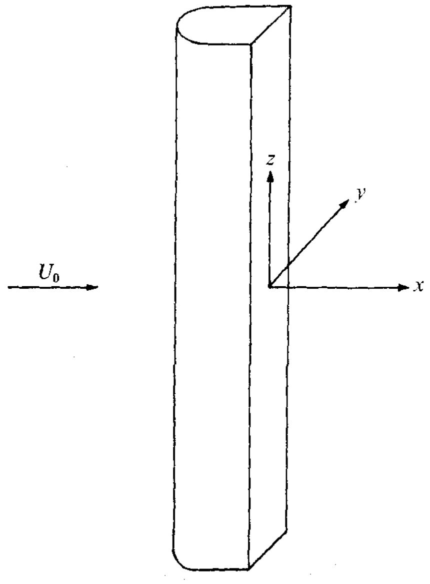

FIGURE 1. D-shaped body and co-ordinate axes.

requirement of high spatial resolution of measurement, calling for the wires to be close to one another, and the need for the wires to be sufficiently far apart to keep errors small in the measurement of the distance and phase difference between them. In preliminary experiments l_0 and z_0 were varied to assess the effects, if any, of interference between the wake of the upstream probe and the flow round the downstream wire. It was found that the power spectra, the phase-difference spectrum and the coherence spectrum of the hot-wire signals were unaffected by changes in these distances save that, as l_0 and z_0 were increased, the phase difference became more variable and the coherence decreased at frequencies other than the shedding frequency. The coherence $\gamma_{rs}(N)$ at the shedding frequency did not vary when l_0 was changed but it decreased slowly as z_0 was increased. Examples of phase and coherence spectra are presented in the next section. In the preliminary experiments it was confirmed that $\theta_{rs}(N)$ was proportional to l_0 for measurements made at a distance $6h$ downstream from the model base along the line $y/h = 0{\cdot}5$. When l_0 was zero, $\theta_{rs}(N)$ was also zero. Since z_0 is small compared with the model span, this result does not establish that the vortices exist in the wake with their axes parallel to the z axis; it does show however that l_0 may be substituted for l in (2) and (3) and the errors resulting from any misalignment between the vortex axes and the z axis will be small. l_0 was measured with the hot wires in position, prior to each set of experiments, using a travelling microscope temporarily placed in the tunnel.

In presenting the results of the phase-angle measurements described here the position of the point in space midway between the two hot wires is taken as the location of the measurement. This will lead to estimates of $\lambda(N)$ and $C(N)$ which are correct only when $\partial\theta_{rs}(N)/\partial(x/h)$ is zero or constant. There will in general be errors in $\lambda(N)$ and $C(N)$ if $\partial^2\theta_{rs}(N)/\partial(x/h)^2$ is non-zero, which will be the case,

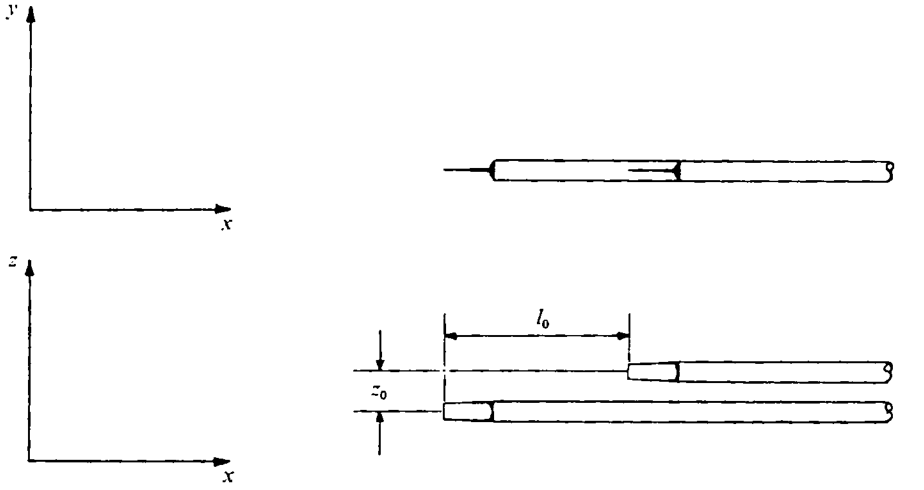

FIGURE 2. Plan and elevation of probe configuration.

for example, when the vortices are moving with an acceleration which is not constant.

Experimental results are presented uncorrected for wind-tunnel blockage. Application of Maskell's (1965) correction leads to an increase in the free-stream velocity U_0 of 1·5 %.

3. Results

The power spectra derived from the two hot-wire signals in the turbulent wake or in the neighbouring fluctuating potential flow were identical in form to one another. The periodic nature of the flow was indicated by a sharp peak in each spectrum at the frequency of vortex shedding N. Except on the wake centre-line, when the peak at N was replaced by one at $2N$, the power levels of these peaks were between 10 and 100 times greater than those at surrounding frequencies. Figures 3 and 4 show typical examples of the coherence and phase spectra obtained. The frequency scale, which is linear, has been non-dimensionalized in each case by being divided by the shedding frequency. There is a peak in the coherence spectrum at the shedding frequency, where the coherence is very close to unity. The phase difference between the hot-wire signals is approximately proportional to the frequency, particularly in the region near the shedding frequency. In view of this and remembering the high coherence at this frequency it was decided that $\theta_{rs}(N)$ should be taken precisely as that given by the phase spectrum. It was thought that this would be more accurate than drawing a straight line through the whole spectrum and taking $\theta_{rs}(N)$ from that. The method of phase-angle measurement may be considered as separating the fluctuating velocities experienced by each hot wire into their sine wave components and measuring the phase difference between the two sine waves at frequency N.

Traverses were made in the x direction at various positions across the wake.

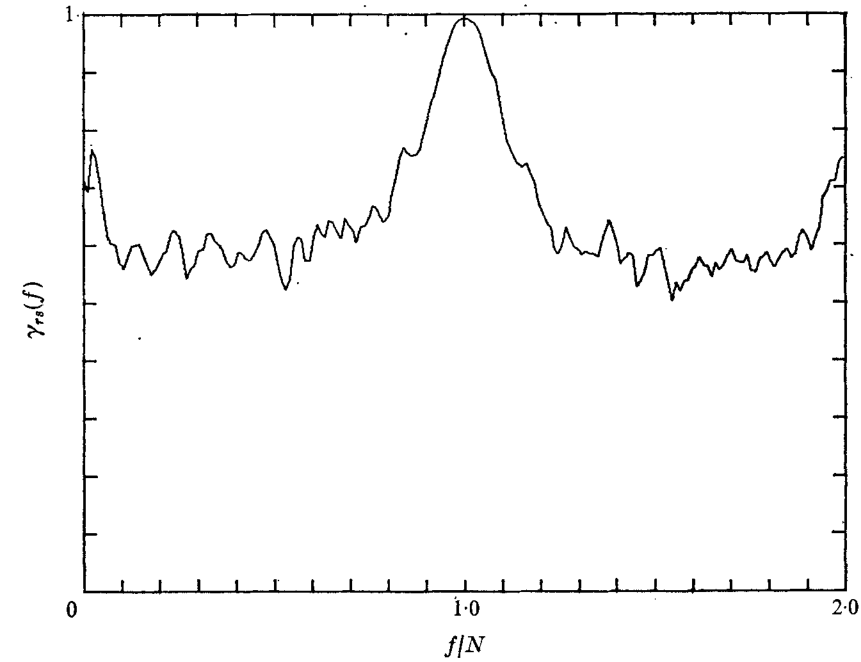

FIGURE 3. Coherence spectrum.

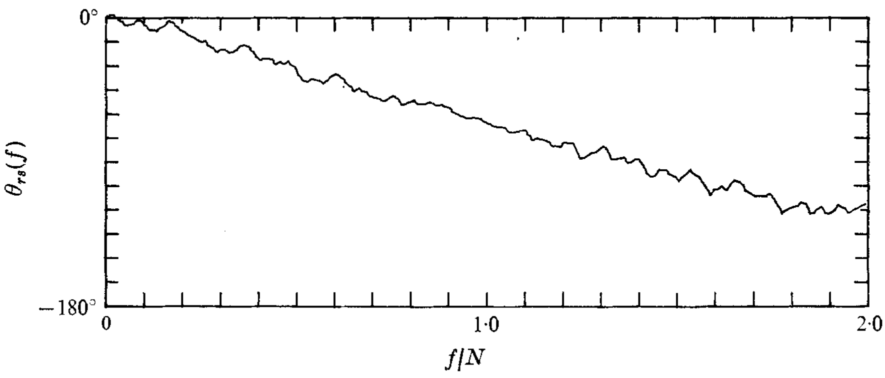

FIGURE 4. Phase spectrum.

Where it was possible to do so these traverses were continued for short distances upstream of the model trailing edge. The first measurements were made along the line $y/h = 0{\cdot}5$. This line is the one along which the shear layer from one side of the body separates and thus can reasonably be expected to be close to the mean path of the centres of the vortices separating from the same side of the body. $\lambda(N)/h$ and $C(N)/U_0$ for $y/h = 0{\cdot}5$ are plotted against x/h in figures 5 and 6. $\lambda(N)/h$ and $C(N)/U_0$ increase strongly between $x/h = 1{\cdot}0$ and $x/h = 3{\cdot}5$ and then more gradually up to the limit of the measurements at $x/h = 12{\cdot}0$. The phase speed increases linearly from $0{\cdot}8U_0$ to $0{\cdot}88U_0$ between $x/h = 3{\cdot}5$ and $x/h = 12{\cdot}0$. Values of U_N given by Bearman (1965) and Jackson (1970) for the wakes of

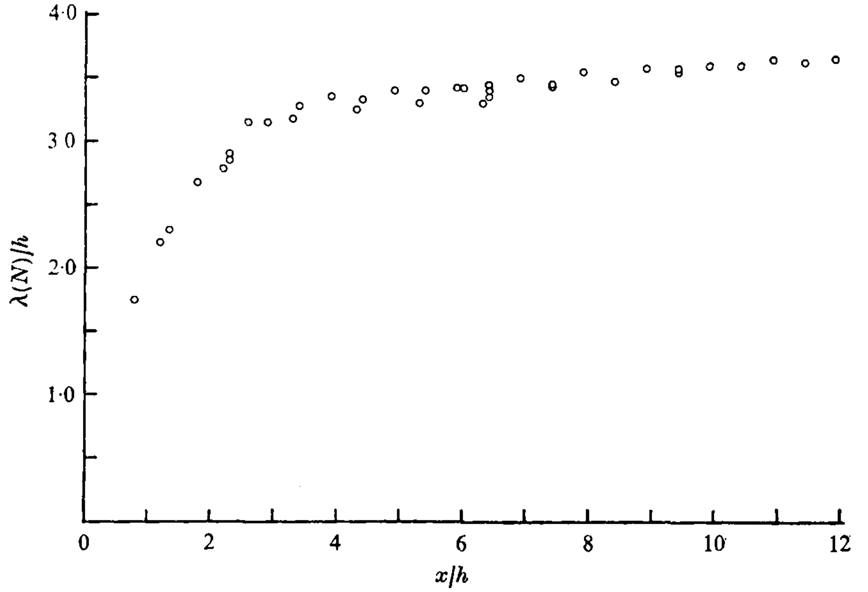

FIGURE 5. $\lambda(N)/h$ *vs.* x/h on $y/h = 0{\cdot}5$.

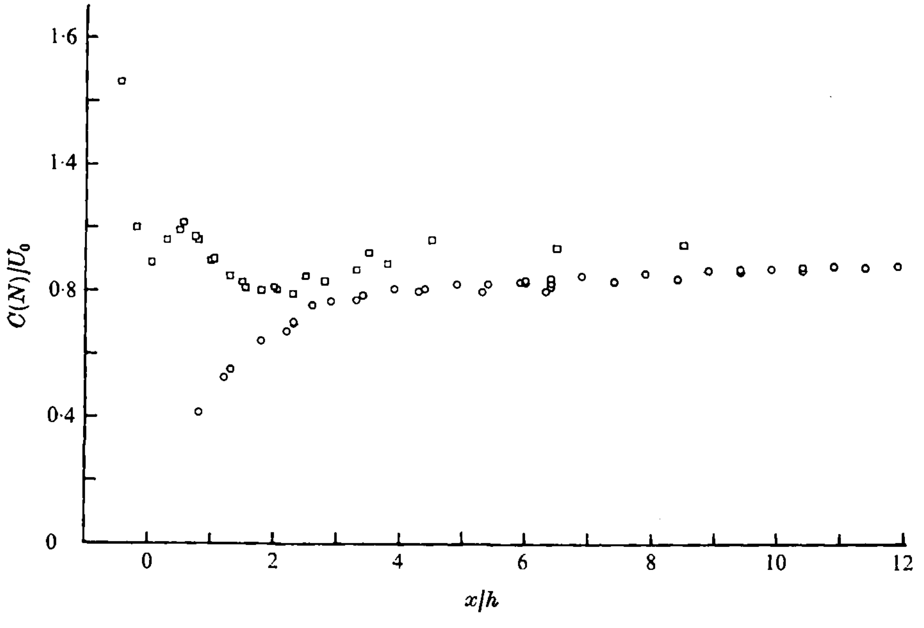

FIGURE 6. $C(N)/h$ *vs.* x/h. ○, $y/h = 0{\cdot}5$; □, $y/h = 1{\cdot}5$.

similar bodies fall within this range. Both these investigators used analog techniques similar to that described in the introduction. Examination of Jackson's results shows that the wavelength of the correlation coefficient increases with distance downstream, indicating a corresponding increase of U_N in a similar way to the increase of $C(N)$ with x/h.

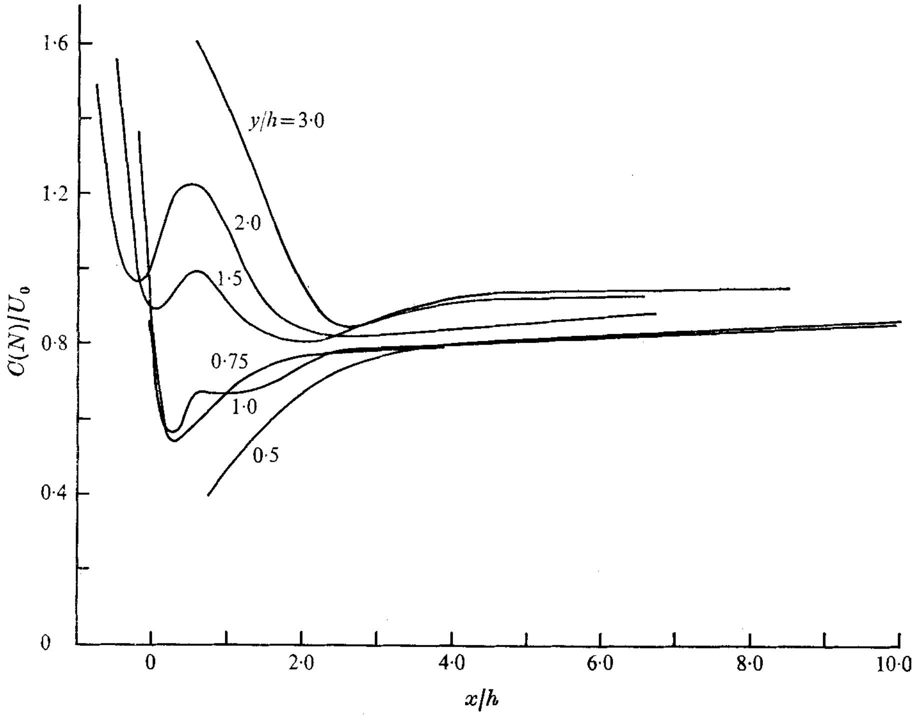

FIGURE 7. Phase-speed measurements.

Further traverses were made along the lines $y/h = 0{\cdot}75$, $1{\cdot}0$, $1{\cdot}5$, $2{\cdot}0$ and $3{\cdot}0$. $C(N)/U_0$ for the traverse along $y/h = 1{\cdot}5$ is shown in figure 6 and mean lines drawn through the experimental points of the various traverses are shown in figure 7. For $x/h \geqslant 3{\cdot}0$ the phase speeds found for $y/h \geqslant 1{\cdot}5$ are noticeably greater than for the traverses made nearer the centre-line of the wake. When the measurements were extended to positions with x/h less than 3·0, $C(N)$ increased for $y/h \geqslant 1{\cdot}5$ and decreased for $y/h \leqslant 1{\cdot}0$. Values of $C(N)$ for $y/h \geqslant 1{\cdot}5$ reach a local maximum at about $x/h = 0{\cdot}5$ and become very large at negative values of x/h. When $y/h = 1{\cdot}0$ and $0{\cdot}75$, the results are similar to those found for $y/h = 0{\cdot}5$ except that $C(N)$ becomes very large as x/h falls below zero. In the region close to the model base the values of $\theta_{rs}(N)$ are very small and this leads to the high values of $C(N)$. For the traverse along $y/h = 0{\cdot}75$, $\theta_{rs}(N)$ was zero at negative values of x/h, implying an infinite value of $C(N)$. Bloor & Gerrard (1966) found that the phase angle between the signals from two hot wires became very small as the wires were moved towards a circular cylinder between 3·5 and 1·5 diameters downstream of its base. The traverse was made along a line 1·5 diameters away from and parallel to the centre-line of the wake. Bloor & Gerrard did not extend their measurements further upstream and did not detect the rise and fall in phase speed found here for the traverses made with $y/h \geqslant 1{\cdot}5$.

In the current experiments phase-angle measurements were also made in a traverse along the wake centre-line $y/h = 0$. In this case the phase speed, which

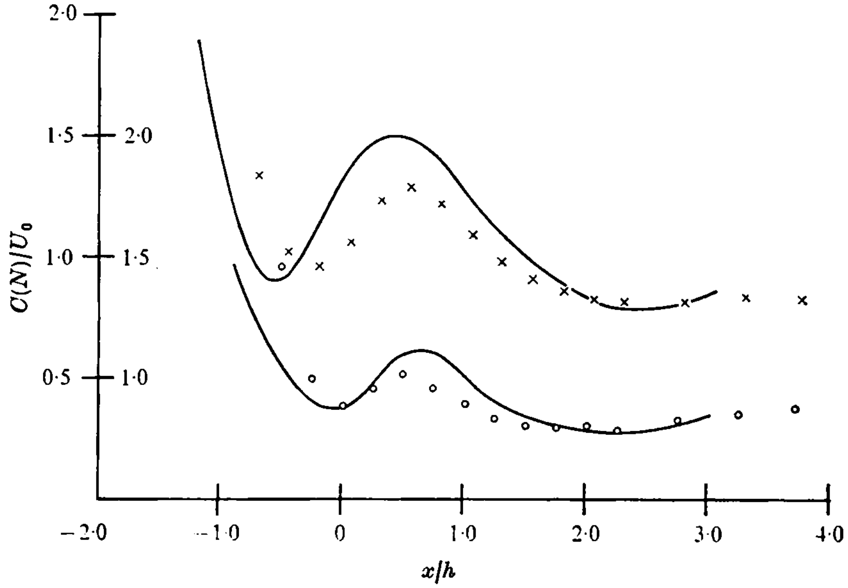

FIGURE 8. Comparison between experimental and numerical phase-speed results. Left-hand phase-speed scale applies to $y/h = 2{\cdot}0$ and right-hand scale to $y/h = 1{\cdot}5$. ——, computed from numerical model of vortex shedding; ○, experiment, $y/h = 1{\cdot}5$; ×, experiment, $y/h = 2{\cdot}0$.

was found at a frequency $2N$, was coincident with $C(N)$ for $y/h = 0{\cdot}5$ for values of x/h greater than 2·0. When x/h was less than 2·0, the phase speed decreased in a similar way to that of $C(N)$ for $y/h = 0{\cdot}5$ along a slightly divergent line.

4. Comparison with a numerical model of vortex shedding

Clements (1973) has produced a numerical model of vortex shedding from a blunt-based body extending to infinity in the upstream direction in an inviscid fluid. In order to investigate the unexpected values of phase speed found in the traverses made with $y/h \geqslant 1{\cdot}5$, Clements' numerical model was used to simulate the author's experiments. In the numerical solution the total velocity was monitored over four cycles of vortex shedding at a series of points representing hot-wire positions along the lines $y/h = 2{\cdot}0$ and $y/h = 1{\cdot}5$. The points were $0{\cdot}6h$ apart, the same distance as the streamwise spacing l_0 of the hot wires in the physical experiment. The phase differences between the velocities at successive points were calculated and the phase speed was found using (3). The results of the computational and wind-tunnel experiments are shown together in figure 8.

The forms of the phase-speed variations with distance downstream predicted by the numerical model and measured experimentally are very similar, although there are differences in magnitude and there is a streamwise displacement of the upstream end. Initially it had been thought that the fluctuation in $C(N)$ with x/h was a nose effect caused by the comparatively short chord of the author's model. This explanation is ruled out because the blunt-based body in the numerical model extends to infinity in the upstream direction.

5. Discussion

The variations in phase speed shown in figure 7 for $x/h < 3{\cdot}0$ and $y/h \geqslant 1{\cdot}5$ cannot be representative of the vortex transport velocity U_N. It follows that the velocity fluctuations experienced by the hot wires at the shedding frequency in the positions corresponding to those measurements are not caused solely by the passage of the vortices in the wake. It is reasonable to suppose that the velocities experienced by the wires are affected by the presence of the model in the flow.

It is well known (see, for example, Gerrard 1965) that the velocity at the outer edge of a boundary layer separating from a bluff body is not constant but fluctuates at the shedding frequency about a mean value U_b which is greater than U_0. The fluctuating velocity of one shear layer is 180° out of phase with that of the other, so that there is an alternating circulation about the body. Weihs (1972), in a theoretical consideration of semi-infinite vortex streets, represented the fluctuating circulation of a bluff body by a bound vortex of alternating sign at the model position. If the induced velocity due to such a bound vortex alone is considered then the phase difference between two sensors, anywhere in a flow assumed to be incompressible, will be zero and the phase speed infinite. In general we may consider that a hot wire in the neighbourhood of a body shedding a vortex wake experiences an induced velocity relative to the stream, fluctuating at the shedding frequency, which is the sum of that due to the motion of the vortex street and that due to an alternating bound vortex at the model position.

It is suggested that as the hot-wire probes are moved downstream of the model trailing edge they pass from a region of flow dominated by the bound vortex to one dominated by the induced velocity due to the passage of the vortex street. This would explain the very high values of the phase speed found upstream of the model trailing edge and the values more representative of U_N found at distances greater than $3h$ downstream. When $x/h > 3{\cdot}0$ the values of $C(N)$ found for $y/h \geqslant 1{\cdot}5$ are higher than those found for $y/h \leqslant 1{\cdot}0$. If the flow around a bluff body can be considered realistically to have the character of that induced by an alternating bound vortex, then the velocities induced in the flow will decay with increasing distance in the same way as those induced by an isolated vortex. For a double row of oppositely signed vortices the velocity induced by the double row as a whole decays more rapidly with distance to a negligible value than does that due to a single vortex whose strength is the same as one of the vortices in the double row. Thus at downstream positions which are farther from the model than from the wake it is still possible for the bound vortex to have a significant effect. This provides a possible explanation for the difference in values of $C(N)$ found for different values of y/h when $x/h > 3{\cdot}0$.

The agreement between the experiments described here and that carried out using the numerical model of vortex shedding shows that the local maxima in $C(N)$ at about $x/h = 0{\cdot}5$ are genuine and not caused by, for example, faulty instrumentation. In the current experiments the mean powers at the vortex shedding frequency were found along each traverse of phase measurements by integrating the areas beneath the power spectral density peaks. The power of turbulence at the shedding frequency was eliminated from these calculations by subtracting the background power from the integrated values. It was found that the power of the velocity fluctuations at frequency N increased by two orders of magnitude between $x/h = 0$ and $x/h = 1{\cdot}0$, a distance comparable with l_0. This rapid increase may be interpreted as an increase in the amplitude of the induced velocity due to the vortex wake while in the same region the amplitude of the induced velocity due to the bound vortex may be expected to decrease. It follows that, in the region of increasing power under consideration, the ratio of the strengths of the induced velocities experienced by one hot wire at any measurement position may be different from that being experienced by the other wire at the same time. In the analysis which follows it is shown that this difference will affect the phase difference between the signals.

The induced velocity of a vortex wake is periodic and if to a first approximation we may consider it to be sinusoidal then a single hot wire placed anywhere in the flow will experience a fluctuating velocity U_F of the form

$$U_F = A \sin \omega t + B \sin (\omega t + \phi), \tag{4}$$

where $A \sin \omega t$ is the induced velocity due to the bound vortex and $B \sin (\omega t + \phi)$ that caused by the passage of the vortex street. A and B are constants which reflect the strengths of the two effects at the position considered and ω is the circular frequency of vortex shedding. ϕ is the phase difference between the two effects. If a second hot wire is placed in the flow it will experience a similar fluctuating velocity of the form

$$U'_F = A' \sin \omega t + B' \sin (\omega t + \phi + \Delta\phi), \tag{5}$$

where $\Delta\phi$ is the phase change caused by the difference in position of the wires. If the wires are separated in the direction in which the vortices are travelling and the bound vortex is fixed in space then $\Delta\phi$ is related directly to the velocity of the wake. It may be shown that the phase difference between U_F and U'_F is given by

$$\Delta\Phi = \tan^{-1}\left(\frac{\sin\phi}{A/B + \cos\phi}\right) - \tan^{-1}\left(\frac{\sin(\phi + \Delta\phi)}{A'/B' + \cos(\phi + \Delta\phi)}\right). \tag{6}$$

With reference to (6), we may suggest that, in the region of rapidly increasing power at the shedding frequency, if the second hot wire is downstream of the first, then A'/B' is much smaller than A/B and hence $\Delta\Phi$ is smaller than it would be if $A'/B' = A/B$. This smaller phase difference would lead to the increased values of the phase speed found in practice.

If $A'/B' = A/B$ and $A \ll B$, then (6) reduces to

$$\Delta\Phi = \Delta\phi, \tag{7}$$

and $\Delta\Phi$ is a true measure of the phase difference between the wires due to the motion of the vortex sheet. If $A'/B' = A/B$ and $B \ll A$ then (6) reduces to

$$\Delta\Phi = 0, \tag{8}$$

and the phase-speed values approach infinity as found for the measurements made upstream of the model base.

The values of $\lambda(N)$ and $C(N)$ found for the traverses along $y/h = 0{\cdot}5$, $0{\cdot}75$ and $1{\cdot}0$ are coincident for $x/h > 3{\cdot}0$. Because of their proximity to the path of the vortex centres, we may expect the induced velocities of the vortex wake on these three traverses to dominate those due to the bound vortex in all regions except those close to the base of the model. The values of $C(N)$ span published measurements of U_N for the wakes of similar bodies and thus it is suggested that the $\lambda(N)$ and $C(N)$ presented here for $x/h \geqslant 3{\cdot}0$, $y/h \leqslant 1{\cdot}0$ may be taken as good estimates of the vortex longitudinal spacing a and vortex transport velocity U_N.

6. Conclusions

Phase-difference measurements have been made in the wake of a two-dimensional bluff body shedding a turbulent wake. It has been shown that values of the vortex longitudinal spacing a and vortex transport velocity U_N that are in fair agreement with published data may be obtained. a and U_N were found to increase with distance downstream. It was found that it is necessary to ensure that the positions where the phase measurements are made are close to the paths of the vortex centres, otherwise results not representative of the wake properties may be obtained. The results may be explained by considering the induced velocities experienced by hot wires in the flow to be the sum of the induced velocity due to the vortex wake and that due to an alternating bound vortex at the model position.

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